

Name \_\_\_\_\_ Class \_\_\_\_\_ Date \_\_\_\_\_

### Lesson 7-4

### Applications of Linear Systems

<b>Lesson Objective</b> ▼ Write systems of linear equations	<b>NAEP 2005 Strand:</b> Algebra <b>Topics:</b> Equations and Inequalities <b>Local Standards:</b> _____
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#### Key Concepts

##### Methods for Solving Systems of Linear Equations

- Graphing** Use graphing for solving systems that are easily graphed. If the point of intersection does not have integers for coordinates, find the exact solution by using one of the methods below or by using a graphing calculator.
- Substitution** Use substitution when one variable has a coefficient of 1 or -1.
- Elimination** Use elimination for solving any system.

#### Examples

**1 Writing Systems** A chemist has one solution that is 50% acid. She has another solution that is 25% acid. How many liters of each type of acid solution should she combine to get 10 liters of a 40% acid solution?

**Define** Let  $a$  = volume of the 50% solution.  
 Let  $b$  = volume of the 25% solution.

**Relate** volume of solution      amount of acid  
**Write**  $a + b = 10$        $0.5a + 0.25b = 0.4(10)$

**Step 1** Choose one of the equations and solve for a variable.

$$a + b = 10$$

Solve for  $a$ .

$$a = 10 - b$$

Subtract  $b$  from each side.

**Step 2** Find  $b$ .

$$0.5a + 0.25b = 0.4(10)$$

$$0.5(10 - b) + 0.25b = 0.4(10)$$

Substitute  $10 - b$  for  $a$ . Use parentheses.

$$5 - 0.5b + 0.25b = 0.4(10)$$

Use the Distributive Property.

$$5 - 0.25b = 4$$

Simplify.

$$-0.25b = -1$$

Subtract  $5$  from each side.

$$b = 4$$

Divide each side by  $-0.25$ .

**Step 3** Find  $a$ . Substitute  $4$  for  $b$  in either equation.

$$a + 4 = 10$$

Solve for  $a$ .

$$a = 6$$

Subtract  $4$  from each side.

To make 10 L of 40% acid solution, you need 6 L of 50% solution and 4 L of 25% solution.

0.50  
0.25  
40% of 10  
0.4 \* 10

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**2 Finding a Break-Even Point** Suppose you have a typing service. You buy a personal computer for \$1,750 on which to do your typing. You charge \$5.50 per page for typing. Expenses are \$0.50 per page for ink, paper, electricity, and other expenses. How many pages must you type to break even?

**Define** Let  $p$  = the number of pages.

Let  $d$  = the amount of dollars of expenses or income.

**Relate**

Expenses are per page expenses plus computer purchase.

Income is price times pages typed.

**Write**

$$d = 0.5p + 1,750$$

$$d = 5.5p$$

Choose a method to solve this system. Use substitution since it is easy to substitute for  $d$  with these equations.

$$d = 0.5p + 1,750 \quad \text{Start with one equation.}$$

$$\begin{array}{r} 0.5p \\ 5.5p \\ \hline 5p = 1,750 \\ p = 350 \end{array}$$

Substitute  $5.5p$  for  $d$ .

Solve for  $p$ .

To break even, you must type 350 pages.

$$\begin{aligned} d &= 0.5p + 1,750 \\ d &= 5.5p \end{aligned}$$

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**Quick Check**

1. Suppose a chemist combines a 25% acid solution and a 50% acid solution to make 40 L of 45% acid solution. How many liters of each solution did she use?

$$\begin{aligned} a + b &= 40 \\ 0.25a + 0.5b &= 0.45(40) \\ 0.25(40 - b) + 0.5b &= 18 \\ 10 - 0.25b + 0.5b &= 18 \\ -18 + 0.25b &= 18 - 10 \\ 0.25b &= 8 \\ b &= 32 \\ a &= 40 - b \\ a &= 8 \end{aligned}$$

$$\begin{aligned} a + b &= 40 \\ a + 32 &= 40 \\ a &= 8 \end{aligned}$$

2. Suppose an antique car club publishes a newsletter. Expenses are \$0.35 for printing and mailing each copy, plus \$770 total for research and writing. The price of the newsletter is \$0.55 per copy. How many copies of the newsletter must the club sell to break even?

$$\begin{aligned} c &= \text{copies} \\ m &= \text{money} \\ m &= 0.55c \\ m &= 770 + 0.35c \\ 0.55c &= 770 + 0.35c \\ -0.35c & \quad -0.35c \\ 0.20c &= 770 \\ c &= 3,850 \end{aligned}$$

8L 25% acid  
32L 50% acid

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$$\begin{aligned} 0.20c &= 770 \\ c &= 3,850 \text{ copies} \end{aligned}$$

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**Example**

**3 Finding Speed** Suppose it takes you 6.8 hours to fly about 2,800 miles from Miami, Florida, to Seattle, Washington. At the same time, a friend flies from Seattle to Miami. His plane travels with the same average airspeed, but his flight only takes 5.6 hours. Find the average airspeed of the planes. Find the average wind speed.

**Define** Let  $A$  = the airspeed. Let  $W$  = the wind speed.

**Relate** with tail wind (rate)(time) = distance  $(A + W)(\text{time}) = \text{distance}$   
 with head wind (rate)(time) = distance  $(A - W)(\text{time}) = \text{distance}$

**Write**  $(A + W) 5.6 = 2800$   $(A - W) 6.8 = 2800$

Solve by elimination. First divide to get the variables on the left side of each equation with coefficients of 1 or -1.

$(A + W) 5.6 = 2,800 \rightarrow A + W = 500$  Divide each side by 5.6

$(A - W) 6.8 = 2,800 \rightarrow A - W = 412$  Divide each side by 6.8

**Step 1** Eliminate  $W$ .

$A + W = 500$

$A - W = 412$

Add the equations to eliminate  $W$ .

$2A + 0 = 912$

**Step 2** Solve for  $A$ .

$A = 456$

Divide each side by 2.

**Step 3** Solve for  $W$  using either of the original equations.

$A + W = 500$

Use the first equation.

$456 + W = 500$

Substitute 456 for  $A$ .

$W = 44$

Solve for  $W$ .

The average airspeed of the planes is 456 mi/h. The average wind speed is 44 mi/h.



Handwritten work showing the division of equations to isolate variables:

$$\frac{(A+W)5.6 = 2800}{5.6} \quad \frac{(A+W)6.8 = 2800}{6.8}$$

$$A+W = 500 \quad A-W = 412$$

**Quick Check**

3. A plane takes about 6 hours to fly you 2,400 miles from New York City to Seattle, Washington. At the same time, your friend flies from Seattle to New York City. His plane travels with the same average airspeed, but his flight takes 5 hours. Find the average airspeed of the planes. Find the average wind speed.

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