10.1 Triangles

the interior angles on a △ always add up to 180°

Goal: Solve problems involving triangles.

Example 1 *Classifying a Triangle by Angle Measures*

In the diagram, $m\angle ABC = 44°$ and $m\angle BAC = m\angle BCA$. Find $m\angle BAC$ and $m\angle BCA$. Then classify $\triangle ABC$ by its angle measures.

angles	side length
right	scalene
obtuse	isosceles
acute	equilateral

You can classify a triangle by its angle measures or by its side lengths. When classified by **angle measures, triangles are acute, right, obtuse, or equiangular.** When classified by side lengths, triangles are equilateral, isosceles, or scalene.

Solution

Let $x°$ represent $m\angle BAC$ and $m\angle BCA$.

$m\angle BAC + m\angle BCA + m\angle ABC = 180°$ Sum of angle measures is 180°.

$\boxed{x} + \boxed{x} + \boxed{44} = 180°$ Substitute values.

$\boxed{2x} + \boxed{44} = 180 - 44$ Combine like terms.

$\dfrac{\boxed{2x}}{2} = \boxed{136}$ Subtract $\boxed{44}$ from each side.

$x = \boxed{68°}$ Divide each side by $\boxed{2}$.

Answer: $m\angle BAC = m\angle BCA = \boxed{68°}$. Because $\angle BAC$, $\angle BCA$, and $\angle ABC$ are $\boxed{\text{acute } \angle\text{'s}}$, $\triangle ABC$ is $\boxed{\text{acute } \triangle}$.

✔ **Checkpoint** Find the value of x. Then classify the triangle by its angle measures.

1.

$3 \cdot 22$

$180°$

$66°$

$3x + 3x + 48 = 180$
$6x + 48 = 180$
$\quad -48 \quad -48$

$\dfrac{6x}{6} = \dfrac{132}{6}$

$\boxed{x = 22 \quad \text{acute}}$

2.

$3 \cdot 30 + 6$
$90 + 6 = 96°$
$96°$
$(3x + 6)°$
$2x° = 60°$
$2 \cdot 30$
$24°$

$2x + 3x + 6 + 24 = 180$
$5x + 30 = 180$
$\quad -30 \quad -30$
$\dfrac{5x}{5} = \dfrac{150}{5}$

$\boxed{x = 30}$

Example 2   *Finding Unknown Side Lengths*

The perimeter of a scalene triangle is 45 inches. The length of the first side is twice the length of the second side. The length of the third side is 15 inches. Find the lengths of the other two sides.

Solution

Draw the triangle. Let x and $2x$ represent the unknown side lengths. Write an equation for the perimeter P. Then solve for x.

10in.

15 in.

$2x$
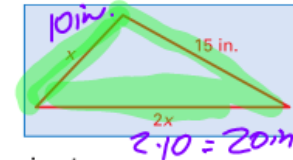
$2 \cdot 10 = 20$ in.

$P = 2x + x + 15$ Formula for perimeter

$45 = 2x + x + 15$ Substitute 45 for P.

$45 = 3x + 15 - 15$ Combine like terms.

$30 = 3x /3$ Subtract 15 from each side.

$10 = x$ Divide each side by 3.

Answer: The length of the second side is 10 inches, and the length of the first side is $2(10) = 20$ inches.

✔ **Checkpoint**   Find the unknown side length of the triangle given the perimeter P. Then classify the triangle by its side lengths.

3. $P = 75$ ft	**4.** $P = 21.6$ m
15 ft 25 ft	X X
x 35ft.	X
$15 + 25 + x = 75$	equilateral
$40 + x = 75$	$x + x + x = 21.6$
$-40 \quad -40$	$\dfrac{3x}{3} = \dfrac{21.6}{3}$
$x = 35 ft.$	$x = 7.2m$
Scalene	
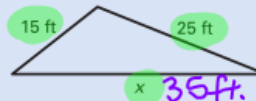
Example 3 — *Finding Angle Measures Using a Ratio*

The ratio of the angle measures of a triangle is $3:4:5$. Find the angle measures. Then classify the triangle by its angle measures.

> For a triangle whose angles measure 50°, 60°, and 70°, you can say that the ratio of the angle measures is 50:60:70, or 5:6:7. Therefore, if you know that the ratio of the angle measures is 5:6:7, you can say that the angle measures are 5x°, 6x°, and 7x° for some value of x.

Solution.

1. Let $3x$, $4x$, and $5x$ represent the angle measures. Write an equation for the sum of the angle measures.

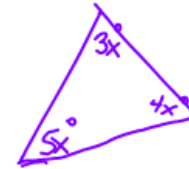$$3x + 4x + 5x = 180° \qquad \text{Sum of angle measures is 180°.}$$
$$12x = 180 \qquad \text{Combine like terms.}$$
$$x = 15 \qquad \text{Divide each side by } 12.$$

2. Substitute 15 for x in the expression for each angle measure.

$$(3 \cdot 15)° = 45° \qquad (4 \cdot 15)° = 60° \qquad (5 \cdot 15)° = 75°$$

Answer: The angle measures of the triangle are $45°$, $60°$, and $75°$. So, the triangle is **acute**.

✓ **Checkpoint**

5. The ratio of the angle measures of a triangle is $7:12:17$. Find the angle measures. Then classify the triangle by its angle measures.

$$7x + 12x + 17x = 180°$$
$$\frac{36x}{36} = \frac{180}{36}$$
$$x = 5$$

$$7 \cdot 5 = 35°$$
$$12 \cdot 5 = 60°$$
$$17 \cdot 5 = 85°$$

acute