10.6 Surface Areas of Pyramids and Cones

Goal: Find the surface areas of pyramids and cones.

Vocabulary

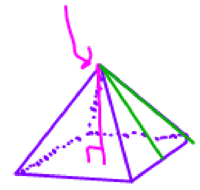
Height of a pyramid: The height of a pyramid is the perpendicular distance between the base and the vertex.

Regular pyramid: In a regular pyramid, the base is a regular polygon and all of the lateral faces of the pyramid are congruent isosceles triangles.

Slant height of a pyramid: The slant height of a pyramid is the height of any of the triangular lateral faces.

Height of a cone: The height of a cone is the distance between the vertex and the center of the base.

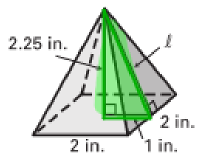
Slant height of a cone: The slant height of a cone is the distance between the vertex and any point on the edge of the base.



Example 1 *Finding the Slant Height of a Pyramid*

In this lesson, all pyramids are regular pyramids.

What is the slant height of the pyramid to the nearest tenth of an inch?



ℓ = Slant height

The slant height ℓ of the pyramid is the hypotenuse of a right triangle. The length of the legs of this triangle are 2.25 inches and 1 inch. Use the Pythagorean theorem to find the slant height.

$a^2 + b^2 = c^2$

$\boxed{2.25}^2 + \boxed{1}^2 = \ell^2$ Pythagorean theorem

$5.0625 + \boxed{1} = \ell^2$ Simplify.

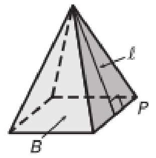
$\sqrt{6.0625} = \sqrt{\ell^2}$ Take positive square root of each side.

$2.5 = \ell$

Answer: The slant height is about $\boxed{2.5 \text{ in}}$.

Surface Area of a Regular Pyramid

Words The surface area S of a regular pyramid is the sum of the base area B and one half of the product of the base perimeter P and the slant height ℓ.

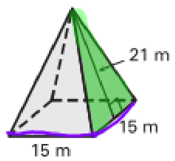


Algebra $S = B + \frac{1}{2}P\ell$

This formula can be used to find the surface area of any regular pyramid.

Example 2 ***Finding the Surface Area of a Regular Pyramid***

Find the surface area of the regular pyramid.



triangles: $\frac{1}{2} \cdot b \cdot h$

$= \frac{1}{2} \cdot 15 \cdot 21$

$= 157.5$

$\times 4$ ← 4 triangles

$\boxed{630 \text{ m}^2}$

base: $15 \cdot 15 = \boxed{225 \text{ m}^2}$

+

855 m^2

1. Find the perimeter and area of the base.

$P = 4(\quad) = \quad$

$B = \quad^2 = \quad$

2. Find the surface area.

$S = B + \frac{1}{2}P\ell$ Write formula for surface area of a pyramid.

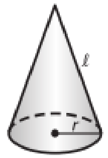
$= \quad + \quad(\quad)(\quad)$ Substitute.

$= \quad$ Simplify.

Answer: The surface area of the pyramid is ______.

Surface Area of a Cone

Words The surface area S of a cone is the sum of the base area B and the product of π, the base radius r, and the slant height ℓ.



Algebra $S = B + \pi r\ell = \pi r^2 + \pi r\ell$

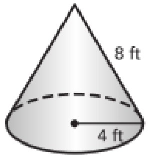
This formula can be used to find the surface area of any cone.

B = area of base   ℓ = slant height

Example 3 *Finding the Surface Area of a Cone*

In your textbook, all cones are right cones, which means that the segment connecting the center of the circular base to the vertex is perpendicular to the base.

Find the surface area of the cone. Round to the nearest square foot.



$S = \pi r^2 + \pi r\ell$ — Write formula for surface area of a cone.

$16\pi + 32\pi$

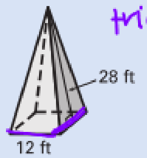
$= \pi(\boxed{4})^2 + \pi(\boxed{4})(\boxed{8})$ — Substitute.

$= \boxed{48}\pi \approx \boxed{150}.79$ — Simplify. Then evaluate using a calculator.

Answer: The surface area of the cone is about $\boxed{150 \text{ ft}^2}$.

Checkpoint Find the surface area of the pyramid or cone. Round to the nearest whole number.

1.



triangles:

$= \frac{1}{2} \cdot b \cdot h$

$= \frac{1}{2} \cdot 12 \cdot 28$

$= 168$

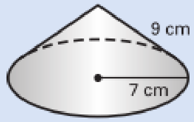
$\times 4 \leftarrow 4$ triangles

672 ft^2

base: $12 \cdot 12 = 144 \text{ ft}^2$

$\boxed{816 \text{ ft}^2}$

2.



$S = \pi r^2 + \pi r \ell$

$= (\pi \cdot 7^2) + (\pi \cdot 7 \cdot 9)$

$= 49\pi + 63\pi$

$= 112\pi$

$\boxed{\approx 352 \text{ cm}^2}$