

9.3 The Pythagorean Theorem

Goal: Use the Pythagorean theorem to solve problems.

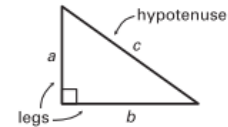
Vocabulary

Hypotenuse:

Legs:

Pythagorean Theorem

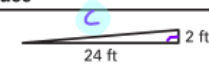
Words For any right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.



Algebra $a^2 + b^2 = c^2$

Example 1 Finding the Length of a Hypotenuse

A building's access ramp has a horizontal distance of 24 feet and a vertical distance of 2 feet. Find the length of the ramp to the nearest tenth of a foot.



$$a^2 + b^2 = c^2 \quad \text{Pythagorean theorem}$$

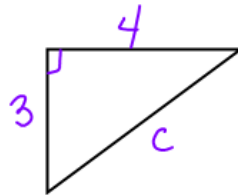
$$2^2 + 24^2 = c^2 \quad \text{Substitute for } a \text{ and for } b.$$

$$4 + 576 = c^2 \quad \text{Evaluate powers and add.}$$

$$\sqrt{580} = c \quad \text{Take positive square root of each side.}$$

$$24.08 \approx c \quad \text{Simplify.}$$

Answer: The length of the ramp is about 24.1 feet.



Example 2 Finding the Length of a Leg

Find the unknown length a in simplest form.

$a^2 + b^2 = c^2$ Pythagorean theorem

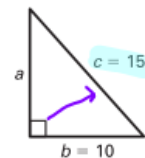
$a^2 + 10^2 = 15^2$ Substitute.

$a^2 + 100 = 225$ Evaluate powers.

$a^2 = 125$ Subtract 100 from each side.

$a = 11.2$ Take positive square root of each side.

$a = 11.2$ Simplify.



Answer: The unknown length a is 11.2 units.

Checkpoint Find the unknown length. Write your answer in simplest form.

<p>1.</p> <p>$a = 6$, $b = 8$, $c = 10$</p> <p>$a^2 + b^2 = c^2$ $6^2 + 8^2 = c^2$ $36 + 64 = c^2$ $100 = c^2$ $10 = c$</p>	<p>2.</p> <p>$c = 13$, $b = 12$</p>	<p>3.</p> <p>$a = 11$, $c = 20$</p>
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$6 \cdot 6 = 36$
 $8 \cdot 8 = 64$

Converse of the Pythagorean Theorem

The Pythagorean theorem can be written in "if-then" form.

Theorem: If a triangle is a right triangle, then $a^2 + b^2 = c^2$.

If you reverse the two parts of the statement, the new statement is called the *converse* of the Pythagorean theorem.

Converse: If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Although not all converses of true statements are true, the converse of the Pythagorean theorem is true.

Example 3 Identifying Right Triangles

Determine whether the triangle with the given side lengths is a right triangle.

a. $a = 8, b = 9, c = 12$

b. $a = 7, b = 24, c = 25$

Solution

a. $a^2 + b^2 = c^2$
 $8^2 + 9^2 \stackrel{?}{=} 12^2$
 $64 + 81 \stackrel{?}{=} 144$
 $145 \neq 144$

b. $a^2 + b^2 = c^2$
 $7^2 + 24^2 \stackrel{?}{=} 25^2$
 $49 + 576 \stackrel{?}{=} 625$
 $625 = 625$

Answer:

not a right triangle

Answer:

yes a right \triangle

✔ **Checkpoint** Determine whether the triangle with the given side lengths is a right triangle.

4. $a = 12, b = 9, c = 15$

5. $a = 10, b = 25, c = 27$