9.3 The Pythagorean Theorem

Goal: Use the Pythagorean theorem to solve problems.
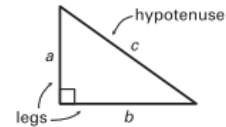

Vocabulary

Hypotenuse:

Legs:

Pythagorean Theorem

Words For any right triangle, the sum of the squares of the lengths of the legs equals the square of the length of the hypotenuse.

Algebra $a^2 + b^2 = c^2$

Example 1 *Finding the Length of a Hypotenuse*

A building's access ramp has a horizontal distance of 24 feet and a vertical distance of 2 feet. Find the length of the ramp to the nearest tenth of a foot.
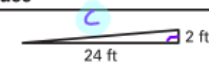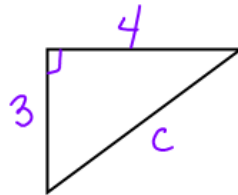
$$a^2 + b^2 = c^2 \quad \text{Pythagorean theorem}$$

$$\boxed{2}^2 + \boxed{24}^2 = c^2 \quad \text{Substitute for } a \text{ and for } b.$$

$$4 + \boxed{576} = c^2 \quad \text{Evaluate powers and add.}$$

$$\sqrt{580} = \sqrt{c^2} \quad \text{Take positive square root of each side.}$$

$$24.08 \approx c \quad \text{Simplify.}$$

Answer: The length of the ramp is about $\boxed{24.1}$ feet.

Example 2   *Finding the Length of a Leg*

Find the unknown length *a* in simplest form.
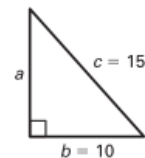
$a^2 + b^2 = c^2$ Pythagorean theorem

$a^2 + \boxed{}^2 = \boxed{}^2$ Substitute.

$a^2 + \boxed{} = \boxed{}$ Evaluate powers.

$a^2 = \boxed{}$ Subtract $\boxed{}$ from each side.

$a = \boxed{}$ Take positive square root of each side.

$a = \boxed{}$ Simplify.

(triangle: a, c = 15, b = 10)

Answer: The unknown length *a* is $\boxed{}$ units.

✔ *Checkpoint*   **Find the unknown length. Write your answer in simplest form.**

(handwritten) $6 \cdot 6 = 36$

$8 \cdot 8 = 64$

1. *(triangle: a = 6, c = ⋮10, b = 8)*

2. *(triangle: a, c = 13, b = 12)*

3. *(triangle: a = 11, c = 20, b)*

(handwritten work under #1:)

$a^2 + b^2 = c^2$

$6^2 + 8^2 = c^2$

$36 + 64 = c^2$

$\sqrt{100} + c^2$

$10 = c$

Converse of the Pythagorean Theorem

The Pythagorean theorem can be written in "if-then" form.

Theorem: If a triangle is a right triangle, then $a^2 + b^2 = c^2$.

If you reverse the two parts of the statement, the new statement is called the *converse* of the Pythagorean theorem.

Converse: If $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Although not all converses of true statements are true, the converse of the Pythagorean theorem is true.

Example 3   *Identifying Right Triangles*

Determine whether the triangle with the given side lengths is a right triangle.

a. $a = 8, b = 9, c = 12$ b. $a = 7, b = 24, c = 25$

Solution

a. $a^2 + b^2 = c^2$

    $\boxed{}^2 + \boxed{}^2 \overset{?}{=} \boxed{}^2$

    $\boxed{} + \boxed{} \overset{?}{=} \boxed{}$

    $\boxed{}\boxed{}\boxed{}$

Answer:

$\boxed{}$

b. $a^2 + b^2 = c^2$

    $\boxed{}^2 + \boxed{}^2 \overset{?}{=} \boxed{}^2$

    $\boxed{} + \boxed{} \overset{?}{=} \boxed{}$

    $\boxed{}\boxed{}\boxed{}$

Answer:

$\boxed{}$

✔ *Checkpoint*   **Determine whether the triangle with the given side lengths is a right triangle.**

4. $a = 12, b = 9, c = 15$	5. $a = 10, b = 25, c = 27$